Indian Statistical Institute
Bangalore Centre
B.Math (Hons.) III Year 2011-2012
Second Semester
Sample Survey and Design of Experiments

Semestral Examination — Date : 7.5.12

Answer at least **three** questions from **each group**.
The maximum you can score is 100.
All notations have their usual meaning. State clearly the results you use.

Group A

1. Suppose a sample of size n is to be drawn from N units using SRSWOR scheme. Let y denote the variable under study and x denote an auxiliary variable.

   (a) Obtain the expressions for the following. (i) $V[\bar{y}]$ and (ii) $Cov[\bar{x}, \bar{y}]$.

   (b) Consider the ratio estimator $\hat{\bar{Y}}_R = (\bar{y}/\bar{x}).\bar{X}$ in this context and explain when it is useful. Show that this estimator is usually biased and the bias is approximately equal to $(1/n - 1/N)\bar{Y}[S_x^2/\bar{X}^2 - S_{xy}/(\bar{Y}\bar{X})]$. When is $\hat{\bar{Y}}_R$ unbiased ?

   [9 + (2+7+1) = 19]

2. In a city a proposal of banning smoking in college campus was being considered. A simple random sample of size 200 was taken without replacement from the population of 2000 colleges. 120 colleges were in favor of the ban, 57 were opposed, and 23 had no opinion.

   (a) Obtain 95% confidence interval for the number of colleges in the population that favored the proposal.

   (b) Do the results of the above sample furnish conclusive evidence that more than half the colleges in the population favored this proposal? Justify stating clearly the assumption you make.

   [5 + 5 =10]

3. Consider a fixed sample size (n) design.

   (a) Prove the following relations between the inclusion probabilities π_i's and π_{ij}'s.

   $$\sum_{i=1}^{N} \pi_i = n, \text{ and } \sum_{j \neq i = 1}^{N} \pi_{ij} = (n-1)\pi_i.$$

   (b) Consider the Horvitz-Thompson estimator $\hat{Y}_{HT} = \sum_{i=1}^{n} y_i/\pi_i$. [Assume $\pi_i > 0\ \forall i$]. Prove that it is unbiased for population total and obtain its variance. Obtain an unbiased estimator for the variance of $\hat{Y}_{HT}$.

   [(4+5)+ (2+ 8 + 2) = 21]

4. An SRSWOR of size $n = n_1 + n_2$ with mean $\bar{y}$ is drawn from a finite population of size N, and a simple random subsample of size n_1 is drawn without replacement from it with mean $\bar{y_1}$. Denote the mean of the remaining n_2 units in the sample by $\bar{y_2}$. Prove the following.

   (a) Cov$\{\bar{y}, \bar{y_1} - \bar{y}\} = 0$,

   (b) $V(\bar{y_1} - \bar{y}) = S^2[(1/n_1) - (1/n)]$ and

   (c) $V(\bar{y_1} - \bar{y_2}) = S^2[(1/n_1) + (1/n_2)]$.

   [Here $S^2 = (1/(N-1)\sum_{i=1}^{N}(Y_i - \bar{Y})^2$.]

   [4 + 4 + 3 = 11]

Group B

5. The effect of 4 different catalysts(A,B,C,D) on the reaction time of a chemical process is being studied. 6 different batches of raw material are used. Due to financial constraints, the following incomplete block design was used. Describe how the data can be analysed so as to see whether the effects of different catalysts differ.

Batch	Catalysts	
1	A	B
2	C	D
3	A	C
4	B	D
5	A	D
6	B	C

[16]

6. In a paper manufacturing factory, the percentage of hardwood concentration in raw pulp and the cooking time of pulp are being investigated for their effects on the strength of the paper. Suppose two concentrations (C) and two cooking times (T) are used and two observations are taken for each level combination. Explain what is meant by (i) the main effects of C and T, (ii) the interaction effect CT and how one can determine them.

[4 x 2 = 8]

7. In an industry an s^m experiment was to be performed, but blocks available were of size s^k only, $(k < m)$.

   (a) Describe a procedure of construction of such a design in which one or more factorial effect are deliberately confounded with blocks.

   (b) Altogether how many effects are confounded ? Justify.

   (c) Consider an effect E not confounded with the block effects. Show that every contrast belonging to E is orthogonal with every contrast belonging to the block effects.

   (c) Take $s = 3, m = 4, k = 2$. Illustrate the procedure of Q(a).

[6 + 3 + 5 + 5 = 19]

8. (a) Consider a (1/4)-replicate of a 2^6 experiment in which the defining relations chosen are $ABCD = I$ and $ABEF = I$. Mention other defining relations if any. Describe the set of treatment combinations used. What are the aliases of the main effects A, E and the interaction CF ?

   (b) Construct an orthogonal array (4,3,2,2).

   Suppose it is used as a fractional replicate of an s^m experiment. What are s and m? What is the fraction? Find the defining relation(s).

[(1 + 3 + 3) + (4 + 2 + 2 + 2) = 17]